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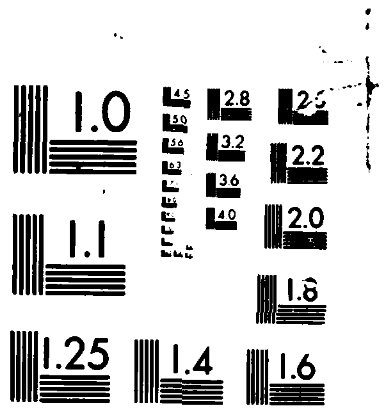
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SURFACE CORRELATION RELATIONSHIPS IN ROUGH SURFACE SCATTERING

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There are a number of approaches to the calculation of the electromagnetic scattering from rough surfaces. These techniques are important in the determination of the effects of the real environment on radar and communications systems. The models contain descriptions of the surface in terms of the statistical distribution of the heights and their degree of correlation. These features are then related to a normalized radar cross section for the terrain, $\sigma°$, through an electromagnetic analysis. In this report we are concerned with questions involving the application of physical optics principles to the analysis. This is important since this approach is used extensively both as a complete solution and as part of composite models. Secondly, we discussed several areas where the statistics and correlation have important roles in determining the scattering from a random rough surface. This is significant in that the usual assumption of a Gaussian distribution of heights with Gaussian correlation may not apply to all types of surfaces. Power spectra

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results suggest that other forms would be more appropriate for some cases.

In the beginning of this report, we examine statistical descriptors of a surface and establish the relationship between them and the conditions for applying a physical optics model to the scattering from that surface. We then examine the sensitivity of the distribution of scattered power to changes in the form of the surface height statistics and correlation function. Specifically, we examine the effect of three different correlation functions on the normalized bistatic cross section of a rough surface.

The results establish, for the first time, a simple condition that allows us to extend the application of physical optics principles to scattering from rough surfaces with other than small slopes. We also show that although there are some differences in the roughness dependence of the diffuse scattered power, the trends tend to be independent of surface height statistics and correlation function.

Preface

The authors wish to acknowledge the contributions of Ms. Margaret Bell (ARCON) who is responsible for the computer programming formulations that provided the results used in this report.

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1. INTRODUCTION

There are a large number of approaches to the calculation of the electromagnetic scattering from rough surfaces.[1-6] The importance of these techniques rests in their relevance to the determination of the effects of the real environment on radar and communications systems. The analyses contain certain assumptions about the nature of the rough surface in relation to the electromagnetic phenomena. Particular emphasis has been placed on the characterization of the surface in terms of the statistical distribution of the heights and their degree of correlation. These features are then related to a normalized radar cross section for the terrain σ° through an electromagnetic analysis.[7,8] We are particularly concerned with questions involving the application of physical optics principles to the electromagnetic analysis. This is important since this approach is extensively used both as a complete solution and as part of composite models.[9]

Since the surface height statistics and the correlation that describes the height relationships for pairs of points on the surface are essential elements of the scattering analysis, it is important to understand how these factors interact in the analytical models. In this report, we will describe several areas where the

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(Due to the large number of references cited above, they will not be listed here. See References, page 27.)

statistics and correlation have important roles in determining the scattering from a random rough surface.

In the first part of this report we will examine statistical descriptors of a surface and establish the relationship between them and the conditions for applying a physical optics model to the scattering from that surface. The second part of the report addresses the question of how sensitive the distribution of scattered power is to changes in the form of the surface height statistics and correlation function. Specifically, we will examine the effect of three different correlation functions on the normalized cross section of a rough surface. For the large scale physical optics cross section, both Gaussian and exponential surface height distributions are considered. The small scale perturbation cross section is also studied as a function of elevation scattering angle for different angles of incidence and different levels of roughness.

2. THE CONDITIONS FOR PHYSICAL OPTICS

Over the years, the arguments to support the validity of physical optics have centered around the use of the Kirchhoff integral representation for the scattered em wave, where the boundary conditions on the surface have been satisfied through the use of the Fresnel plane wave reflection coefficients. The boundary conditions are met by specifying the total field on the rough surface as the sum of an incident field and a scattered field. The scattered field is expressed as the product of the incident field and the Fresnel plane wave reflection coefficient. The use of the Fresnel plane wave reflection coefficients is justified if the local radii of curvature on the rough surface are large compared to a wavelength ($R_c >> \lambda$). We wish to relate this constraint to statistical parameters of the surface.

Ulaby et al[10] give two conditions which must be satisfied for the Kirchhoff approximation to be valid. These are $kT > 6$ and $T^2/\sigma > 2.76\ \lambda$ where $k = 2\pi/\lambda$, λ = em wavelength, T = surface correlation length and σ = standard deviation in surface height. In this report, we will demonstrate that the single condition ($T >> \lambda$) is a sufficient condition to imply that the radius of curvature, R_c, must be large compared to a wavelength ($R_c >> \lambda$), for surfaces with a Gaussian distribution in heights and either a power law correlation function or a Gaussian correlation function. We will also show that, except for the case of a surface with small slopes, this condition is a necessary condition as well. This means that, for small surface slope conditions physical optics can apply even when the correlation

10. Ulaby, F.T., Moore, R.K., and Fung, A.K. (1982) Microwave Remote Sensing, Vol. II, Addison-Wesley, Massachusetts.

length is not large compared to the wavelength. However, if one should want to apply physical optics to surfaces with larger slopes, then the surfaces are limited only to those where $T >> \lambda$.

3. CORRELATION LENGTH AND RADIUS OF CURVATURE

In the previous section, we indicated that the relationships between T and λ and R_c and λ depended on the magnitude of the surface slopes of the scattering surface. The first point in developing the specifics of these relationships is to introduce the surface height correlation functions for which the analysis has been carried out. It should be noted that the surface is assumed to have a Gaussian height distribution function and is homogeneous and isotropic.

The surface is considered to have either of two types of correlation function. The first is the Gaussian correlation:

$$R(\tau) = \sigma^2 \exp(-\tau^2/T^2) \tag{1}$$

where

σ = standard deviation in surface heights

and

τ = distance between two points on the surface.

The second form of correlation is a power law:

$$R(\tau) = \sigma^2 [1 + \tau^2/T^2]^{-k_p} \tag{2}$$

where k_p = a positive number. Cosgriff et al[11] used a power law correlation function with $k_p = 3/2$.

Next, we introduce the expression for the radius of curvature and relate it to the correlation function of the surface. The results are obtained for each of the two correlation function types.

11. Cosgriff, R., Peake, W., and Taylor, R. (1960) Terrain Scattering Properties for Sensor System Design (Terrain Handbook II), Ohio State University, EES Bulletin 181.

The average radius of curvature, R_c of a Gaussian surface described by the equation $z = \xi(x, y)$ is given by

$$R_c = \frac{1}{<|K|>} = \left\langle \left| \frac{[(1 + (z')^2]^{3/2}}{z''} \right| \right\rangle \tag{3}$$

where K is the curvature. The average of $(z')^2$ is given by

$$<(z')^2> = \left\langle \frac{\partial \xi}{\partial x} \frac{\partial \xi}{\partial x} \right\rangle = \left| \frac{\partial^2 R(\tau = 0)}{\partial \tau^2} \right| .$$

Also, the average of $(z'')^2$ is given by

$$<(z'')^2> = \left| \frac{\partial^4 R(\tau = 0)}{\partial \tau^4} \right| . \tag{4}$$

For a Gaussian correlation function,

$$<(z')^2> = \frac{2\sigma^2}{T^2} \tag{5}$$

and

$$<(z'')^2> = \frac{12\sigma^2}{T^4} .$$

For a power law correlation function,

$$<(z')^2> = \frac{2k_p \sigma^2}{T^2} \tag{6}$$

and

$$<(z'')^2> = \frac{12k_p^2 \sigma^2}{T^4} + \frac{12k_p \sigma^2}{T^4} = \frac{12\sigma^2}{T^4} (k_p + 1)k_p .$$

These results relate the properties of the slopes and slope derivatives to derivatives of the surface height correlation function. This is consistent with the theorem from random processes that states that the distribution of the derivative of a normal process with zero mean and variance, σ^2, is again normal and has a variance

$$\sigma_s^2 = -\sigma^2 \rho''(0) . \tag{7}$$

We next use these results to establish our relationships. As discussed before, Ulaby et al[10] arrived at similar conditions under the restriction of small slopes. We consider the entire range of surface slopes, though, and hence can not use their simplified expressions. As a result, the analysis is more complicated.

3.1 General Solution

We have examined the statistical relations describing the variances of the surface height slopes and the slope derivatives. We now extend that concept to the covariance relation between these two quantities. As before, we have

$$\tau = \sqrt{(\Delta x)^2 + (\Delta y)^2} \tag{8}$$

and

$$\langle \nabla^m \xi \nabla^n \xi \rangle = (-1)^{m+n} \frac{\partial^{m+n} R(\tau)}{\partial x^{m+n}} .$$

We now evaluate $\left\langle \frac{\partial \xi}{\partial x} \frac{\partial^2 \xi}{\partial x^2} \right\rangle$ for the cases of Gaussian and power law correlation functions for the surface heights. For the Gaussian case, we have

$$(-1)^3 \frac{\partial^3 R(\tau)}{\partial x^3} \Bigg|_{\tau=0} = (-1) \left[\frac{12 R(\tau)(x - x_o)}{T^4} - \frac{8 R(\tau)(x - x_o)^3}{T^6} \right] \Bigg|_{\tau=0} = 0 . \tag{9}$$

Similarly, for the power law case

$$(-1)^3 \frac{\partial^3 R(\tau)}{\partial x^3} \Bigg|_{\tau=0} = (-1) \left[\frac{12 k_p (k_p + 1) R(\tau)(x - x_o)}{T^4 [1 + (\tau/T)^2]^2} - \frac{8 k_p (k_p + 1) R(\tau)(x - x_o)^3}{T^6 [1 + (\tau/T)^2]^3} - \frac{8 k_p (k_p + 1)^2 R(\tau)(x - x_o)^3}{T^6 [1 + (\tau/T)^2]^3} \right] \Bigg|_{\tau=0} = 0 . \tag{10}$$

Thus, for a Gaussian surface (the heights have a Gaussian distribution) with either a Gaussian or power law correlation among the heights, the slope and slope derivatives at any point on the surface are statistically uncorrelated. Further, the previously cited theorem from random processes can be applied to get the result for the probability density function describing the slope at a point:

$$p(z') = [2\pi <(z')^2>]^{-1/2} \exp \left\{ - \left(\frac{(z')^2}{2<(z')^2>} \right) \right\} . \tag{11}$$

Also, for the slope derivative, we have

$$p(z'') = \frac{1}{\sqrt{2\pi} <(z'')^2>^{1/2}} \exp \left[\frac{-(z'')^2}{2<(z'')^2>} \right] . \tag{12}$$

Then, since z' and z'' are uncorrelated we can write a joint density function p(z', z'') as

$$p(z', z'') = [4\pi^2 <(z')^2><(z'')^2>]^{1/2} \exp \left\{ -1/2 \left(\frac{(z')^2}{<(z')^2>} + \frac{(z'')^2}{<(z'')^2>} \right) \right\} . \tag{13}$$

Then, from Eq. (3) we can write the equation for the expected value of the curvature as:

$$<|K|> = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |z''| \left[|1 + (z')^2| \right]^{-3/2} p(z', z'') dz' dz'' . \tag{14}$$

Integrating over z'' then yields

$$<|K|> = \sqrt{2/\pi} \sqrt{<z''^2>} \frac{2}{\sqrt{2\pi} \sqrt{<(z')^2>}} \int_0^{\infty} [1 + (z')^2]^{-3/2} \exp \left[\frac{-(z')^2}{2<(z')^2>} \right] dz' . \tag{15}$$

After some algebraic manipulation, we obtain

$$<|K|> = \left(\sqrt{<(z'')^2>/<(z')^2>\pi} \right) U(1/2,\ 0,\ 0.5<(z')^2>^{-1}) \tag{16}$$

where U(a, b, x) is the confluent hypergeometric function of the second kind with $x = 0.5<(z')^2>^{-1}$ (see Reference 12).

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12. Abramowitz, M., and Stegun, I. A. (Ed.) (1964) Handbook of Mathematical Functions With Formulas, Graphs and Mathematical Tables, NBS Applied Mathematics Series No. 55.

3.2 Different Slope Regimes

Now, having obtained this analytic expression for $<|K|>$ (or R_c), we want examine the relationships between R_c and σ and T for various slope regimes.

As a first step we examine the small slope regime using the general solution. An asymptotic expansion of U(a, b, x) in terms of the inverse of the slope leads to a zero order solution

$$<|K|> \approx \left(\sqrt{<(z'')^2>/\pi}\right)\left(\sqrt{<(z')^2>}\right)^{-1}\left(1/(2<(z')^2>)\right)^{-1/2} \approx \sqrt{2<(z'')^2>/\pi} . \tag{17}$$

The relations for $<(z'')^2>$ for the two correlation functions then give:

$$<|K|> \approx 2.76\,\sigma/T^2 \qquad \text{(Gaussian)}$$

and

$$<|K|> \approx 2.76\,\sigma\sqrt{k_p(k_p+1)}/T^2 \qquad \text{(Power Law)} . \tag{18}$$

These results are equivalent to the ones obtained by Ulaby et al[10] for small slope. Then, from $\sigma/T << 1$, we have $R_c >> 0.4T$ for Gaussian cases and $R_c >> 0.4T/\sqrt{k_p(k_p+1)}$ for power law correlation. These results yield the sufficient condition that $T >> \lambda$ implies $R_c >> \lambda$ for Gaussian correlation and for any power law case where $k_p \sim 1$ (including the 3/2 law used by Cosgriff et al.[11]). One particular point should be made here. The case where $T \approx \lambda$ also leads to the results $R_c >> \lambda$ for the Gaussian case (and a similar argument would apply for some power law cases). Thus, for the small slope regime we have the overall result that $T >> \lambda$ is indeed a sufficient condition for physical optics models to apply but it is not a necessary condition. As long as the surface slopes are small, even surfaces with only a small degree of correlation ($T \approx \lambda$) can satisfy this condition.

For small slopes, the asymptotic expansion of U(a, b, x) to first order gives the results:

$$<|K|> = [2.76\,\sigma/T^2]\;[1 - 3/2(\sigma/T)^2] \tag{19}$$

for Gaussian correlation, and

$$<|K|> = [2.76\,\sigma/T^2]\;\sqrt{k_p(1+k_p)}\;[1 - 3/2k_p(\sigma/T)^2] \tag{20}$$

for Power Law correlation.

We have already showed the connection between $T >> \lambda$ and $R_c >> \lambda$ for the zeroth order solution. For the Gaussian first order result we have

$$R_c \approx (T^2/2.76\ \sigma)\ (1 + 3/2(\sigma/T)^2)$$

or

$$R_c > T^2/(2.76\ \sigma) \Longrightarrow R_c >> T$$

and hence

$$T >> \lambda \Longrightarrow R_c >> \lambda \text{ for } \sigma/T << 1\ . \tag{21}$$

The analysis is similar for the power law case. Thus, the general solution gives $T >> \lambda$ as a sufficient condition for $R_c >> \lambda$ for both correlations when $\sigma/T << 1$. As in the zero order result, $T \approx \lambda$ also gives $R_c >> \lambda$ so the condition is still not a necessary one.

The next solution for the general result is that in the large slope regime; the intermediate slope region is deferred to the end since the simplifying solutions do not apply there.

For large slope conditions ($\sigma/T >> 1$), $(2 <z'^2>)^{-1} \to 0$ and

$$<|K|> \approx (2/\pi)\ \sqrt{<z''^2>/<z'^2>} + 0[(1/2<z'^2>)\ln(1/2<z'^2>)] \tag{22}$$

or

$$R_c \approx \pi/2\ \sqrt{<(z')^2>/<(z'')^2>}\ .$$

We now introduce the results for the two correlations and get

$$R_c \approx 0.6T \quad \text{(Gaussian)} \qquad \text{and} \qquad R_c \approx 0.6T/\sqrt{k_p} \quad \text{(Power Law)}\ . \tag{23}$$

As a result we have that $T >> \lambda$ is both sufficient and necessary for $R_c >> \lambda$ for the Gaussian case where $\sigma/T >> 1$. In addition, as long as the restriction $k_p \approx 1$ holds, this is also true for power law correlation functions.

Finally, we address the most complicated case where we have to evaluate the function explicitly for various intermediate slope cases. These generally correspond to intermediate values of the argument X but there is some overlap into the asymptotic regimes. Procedures for evaluating the function U(1/2, 0, X) are outlined in Abramowitz.[12] To see how R_c varies in the intermediate X range we consider several cases as $10.0 \geqq X \geqq 0.1$. Over this range, for the Gaussian case, $1.5 \geqq (\sigma/T) \geqq 0.16$, and the evaluation of $U(1/2, 0, X)$ leads to the result $0.5T \leqq R_c \leqq 2.5T$. To complete the correspondence between intermediate ranges of slopes and X-values we must include the small X solution for $0.1 \geqq X \geqq 0.0025$. Here $R_c = 0.6T$. Similarly, the large X solution for $1.0 \leqq \sigma/T \leqq 10$ results in $2.3T \leqq R_c \leqq 3.6T$. The remaining intermediate range results are summarized in Table 1. Thus, over the entire range of intermediate σ/T values, R_c is of the same order as T and hence in that range $T >> \lambda \rightarrow R_c >> \lambda$ and $R_c >> \lambda \rightarrow T >> \lambda$. Thus $T >> \lambda$ is both necessary and sufficient for $R_c >> \lambda$ when we have intermediate σ/T values and Gaussian correlation.

Table 1. Results for R_c in the IIntermediate Range of σ/T Values

X	σ/T	$<\|K\|>$	R_c
10	0.16	0.4/T	2.5T
3	0.3	0.5/T	2.1T
1.6	0.4	0.7/T	1.5T
1	0.5	0.9/T	1.1T
0.5	0.7	1.1/T	0.9T
0.4	0.8	1.3/T	0.8T
0.1	1.5	1.9/T	0.5T

For the power law case the tabular results are similar with σ/T including a $\sqrt{k_p}$-term and R_c having a $\sqrt{k_p + 1}$-term. Thus, as long as $k_p \approx 1$, the necessary and sufficient relationship applies to this correlation case as well.

At this point we have established that there is a direct relationship between the correlation length of a surface and the radius of curvature as far as the justification for the use of a physical optics model is concerned. We have shown that, except for small slopes, the class of surfaces for which $R_c >> \lambda$ is restricted to those for which $T >> \lambda$.

One additional comment is worth mentioning. It is well known that as the average surface slopes become large ($\sigma/T \gtrless 1$), the physical optics current does not truly describe the behavior of the surface currents. Shadowing becomes important under those conditions. The traditional approach to the current is to modify the cross section obtained from physical optics by a shadowing correction factor S, which describes how much of the surface is unlit, that is, where $J \to 0$ in the shadow regions.[13] There has been discussion as to the validity of this approach.

Having shown that the surface height correlation is an essential criterion for establishing the validity of applying physical optics principles to the analysis of rough surface scattering, we now turn to the second theme of the report. We examine how use of particular surface statistics and correlation functions affect the actual determination of the scattering for a range of surface roughness levels.[14]

4. CORRELATION FUNCTION AND SCATTERING

In this section we are concerned with the changes that occur in the distribution of the scattering from a rough surface as a function of roughness when alternative forms of surface height statistics and correlation function are used in the calculation. The study is carried out in terms of the normalized scattering cross section of the surface, $\sigma°$. The variation of $\sigma°$ with angle, roughness and correlation function is examined for different surface height distributions. The primary concern is the regime where $T >> \lambda$ and physical optics models can be used. For completeness, the results when the surface heights and correlation lengths are appropriate to perturbation method analysis are also considered.

Three basic forms for the correlation function are assumed in this section:

(1) The Gaussian correlation function, given by Eq. (1),

(2) The Bessel function correlation function

$$R(\tau) = \sigma^2[1 + 1/8(\tau/T)^2](\tau/T)K_1(\tau/T) - \sigma^2(\tau/T)^2K_0(\tau/T), \tag{24}$$

(3) The power law correlation function, given by Eq. (2).

Because of its tractability in analytical expressions for $\sigma°$, the Gaussian form has been widely used by many authors over the years (Beckmann and Spizzichino,[1] Ruck et al,[2] Ulaby et al[10]). However, not all terrain nor sea states can be represented accurately by a Gaussian correlation function, because the Gaussian correlation function has a finite dc component in the power spectral domain (at $k = 0$ where k = wavenumber). The Bessel function $R(\tau)$, form has been used

13. Sancer, M.I. (1969) Shadow-corrected electromagnetic scattering from a randomly rough surface, IEEE Trans. Antennas and Propag., AP-17:577-585.

14. Brown, G.S. (1984) The validity of shadowing and corrections in rough surface scattering, Radio Science 14, 6:1461-1468.

by Miller et al[15] to represent the sea surface. It is not analytic at $\tau = 0$, so that in the power spectral domain S(k), an upper limit to the wavenumber $k = k_c$ should be introduced, to give a more physically meaningful correlation function. The power law correlation function form was introduced by Cosgriff et al.[11] It was first used to represent asphalt roads. The power spectral density (where k is the wave number) is given by

$$S(k) = \left(\frac{2\sigma^2}{\pi}\right) \int_0^\infty \tau \; c(\tau) \; J_o(k\tau) d\tau \tag{25}$$

where $c(\tau) \equiv R(\tau)/\sigma^2$ and $J_o(k\tau)$ is the zero order Bessel function of the first kind. The form of the resultant power spectral density is often a strong motivation for selecting a particular surface correlation function. The power spectral densities corresponding to each of the correlation functions given above are

(1) Gaussian

$$S(k) = \left(\frac{T^2\sigma^2}{\pi}\right) \exp[-(1/4)\, T^2 k^2] \, , \tag{26}$$

(2) Bessel function

$$S(k) = \frac{12\, k^4 T^6 \sigma^2}{\pi (k^2 T^2 + 1)^4} \tag{27}$$

(3) Power Law

$$S(k) = (2\, T^2 \sigma^2 / \pi) \exp[-kT] \, . \tag{28}$$

For the large scale surface roughness, which is based upon physical optics, the normalized scattering cross section $\sigma°$ is given by the expression

$$\sigma° = |\beta_{pq}|^2 J \tag{29}$$

where

β_{pq} = matrix elements for linear polarization states (see Ruck et al[2]), and

J = a function dependent upon the surface height statistics and surface correlation function.

15. Miller, L.S., Brown, G.S., and Hayne, G.S. (1972) Analysis of Satellite Altimeter Signal Characteristics and Investigation of Sea-Truth Data Requirements, NASA-CR-137465.

In order to simplify the analysis of this report, we note that the cross section scattering matrix elements do not depend on the surface statistics or correlation. They include just polarization dependent elements and geometrical factors. Thus, the β_{pq} contributions can be eliminated when examining the surface statistical dependencies of $\sigma°$. In the remainder of this section, we then will be concerned only with the changes in J that result from varying the form of the correlation function and the statistical distribution of the surface heights.

4.1 Forms of the J-Term

For Gaussian distributed surface heights, Beckmann and Spizzichino[1] give the result:

$$J_{GA} = \left(\frac{8\pi^2}{\lambda^2} \right) \int_0^\infty J_o(v_{xy}\tau) [\exp[-\Sigma^2(1 - c(\tau))] - e^{-\Sigma^2}] \tau d\tau \tag{30}$$

where

λ = em wavelength,

$v_{xy} = \sqrt{v_x^2 + v_y^2}$,

$v_x = \left(\frac{2\pi}{\lambda} \right) (\sin\theta_i - \sin\theta_s \cos\phi_s)$,

$v_y = \left(\frac{2\pi}{\lambda} \right) (\sin\theta_s \sin\phi_s)$,

θ_i = elevation angle of incidence,

θ_s = elevation angle of scattering,

ϕ_s = azimuthal angle of scattering,

Σ = Rayleigh parameter, and

$= \left(\frac{2\pi\sigma}{\lambda} \right) (\cos\theta_i + \cos\theta_s)$.

For exponentially distributed surface heights, Brown[16] derived an equivalent result

$$J_{EX} = J_D + J_S \tag{31}$$

16. Brown, G.S. (1982) Scattering from a class of randomly rough surfaces, Radio Science, 17(No. 5):1274-1280

where

$$J_D = \left(\frac{4\pi}{\lambda^2}\right)(2\pi)\int_0^\infty J_o(v_{xy}\tau)[(1+2/3\,\Sigma^2(1-c(\tau)))^{-3/2} - (1+2/3\Sigma^2)^{-3/2}]\tau d\tau \tag{32}$$

and

$$J_S = \frac{4\pi A}{\lambda^2}\,[(1+2/3\,\Sigma^2)^{-3/2} - (1+1/3\,\Sigma^2)^{-3}]\,\delta(v_x^{SPEC})\,\delta(v_y^{SPEC})\,. \tag{33}$$

Here, A = unit area of rough surface. The J_D term given by Eq. (32) represents the diffuse power scattered into all directions, whereas the J_S term given by Eq. (33) represents an additional amount of diffuse power scattered into the specular direction (Brown[16]).

These expressions represent the general form of the solution. In order to examine the effects of using different types of correlation, we must replace $c(\tau)$ in Eq. (31) and Eq. (32) by the three explicit forms that we wish to study. These are given by Eqs. (1), (2), and (24). In the next section we will show how the successive correlation functions affect the roughness dependence of surfaces with Gaussian height distributions. Then, we will consider the effect of surface type by comparison with similar results for surfaces with exponential height distributions.

4.2 Correlation Function and Surface Roughness

In this section we present results for the changes in rough surface scattering, as surface roughness increases. The effect of using different correlation functions is shown. The roughness is introduced in terms of the Rayleigh roughness parameter, Σ. The surfaces are chosen to satisfy physical optics conditions. The results are not limited to any particular surface type since the dependence on dielectric constant has been excluded by making comparisons in terms of the quantity J defined in Eq. (29). Results are given for azimuth angle $\phi_S = 0°$. The relative breadth of the scattering pattern is shown by the amount scattered for successive values of θ_S. The surface correlation length is taken to be T = 15 m. The wavelength is $\lambda = 0.275$ m. The incident elevation angle is $\theta_i = 85°$. Although the results presented in this report are all at this incident angle, additional calculations were performed at small and intermediate θ_i, and indicate that the results are similar for all cases. Then the results are given for Σ_1 corresponding to $\sigma_1 = 3.16$ m; Σ_2 for $\sigma_2 = 0.316$ m; and Σ_3 for $\sigma_3 = 0.0316$ m.

4.2.1 RESULTS FOR GAUSSIAN SURFACE HEIGHTS

Table 2 shows the scattering cross section pattern for a Gaussian surface for large Rayleigh parameter, Σ_1. Here, c_1 is the Gaussian correlation case, c_2 is the Bessel case and c_3 is the power law case. The table shows a broad pattern increasing in magnitude at large θ_s for all three forms. The Gaussian and power law results are similar and tend to exceed the Bessel case except at small θ_s. Table 3 shows the results for intermediate Rayleigh parameter Σ_2. Here, the overall pattern has narrowed somewhat but the trend is the same as in Table 2. Table 4 shows the results for small Rayleigh parameter, Σ_3. Here the contributions are mainly in directions close to specular.

Table 2. Results for Gaussian Surface With Large Roughness

θ_s	$J_{GA}(c_1)$	$J_{GA}(c_2)$	$J_{GA}(c_3)$
0	0.169	1.251	0.5
5	0.365	1.395	0.9
10	0.714	1.555	1.4
15	1.287	1.740	2.2
20	2.176	1.959	3.1
25	3.495	2.224	4.4
30	5.392	2.553	6.0
35	8.061	2.972	8.1
40	11.77	3.517	10.8
45	16.93	4.249	14.5
50	24.14	5.261	19.4
55	34.42	6.716	26.3
60	49.51	8.910	36.2
65	72.73	12.427	51.4
70	110.90	18.554	76.4
75	180.3	30.566	122.1
80	328.2	58.867	220.0
85	745.2	151.453	499.0
90	3007.0	820.422	2048.0

Table 3. Results for Gaussian Surface With Intermediate Roughness

θ_s	$J_{GA}(c_1)$	$J_{GA}(c_2)$	$J_{GA}(c_3)$
0	$< 10^{-7}$	0.008	< 0.1
5	$< 10^{-7}$	0.011	< 0.1
10	$< 10^{-7}$	0.016	< 0.1
15	$< 10^{-7}$	0.024	< 0.1
20	$< 10^{-7}$	0.038	< 0.1
25	$< 10^{-7}$	0.063	< 0.1
30	$< 10^{-7}$	0.108	< 0.1
35	$< 10^{-7}$	0.196	< 0.1
40	$< 10^{-7}$	0.382	< 0.1
45	$< 10^{-7}$	0.801	< 0.1
50	$< 10^{-7}$	1.828	< 0.1
55	$< 10^{-7}$	4.595	< 0.1
60	61×10^{-7}	12.889	< 0.1
65	0.008710	41.187	0.6
70	3.146	155.339	27.1
75	308.6	743.807	657.9
80	9313.0	5,501.954	8,649.0
85	60,690.0	10,464.87	97,600.0
90	23,245.0	18,948.55	19,450.0

Combining these three sets of results confirms that for all three correlation functions the patterns are similar to those of Beckmann and Spizzichino.[1] For smooth surfaces, scattering tends to be in the specular direction and the scattering pattern broadens as the roughness level of the surface increases. It should be noted that the specular point null for Bessel correlation in Table 3 is present only for $\theta_i = 85°$. There is no similar result at other θ_i values.

Table 4. Results for Gaussian Surface With Small Roughness

θ_s	$J_{GA}(c_1)$	$J_{GA}(c_2)$	$J_{GA}(c_3)$
0	$< 10^{-7}$	< 0.001	< 0.1
5	$< 10^{-7}$	< 0.001	< 0.1
10	$< 10^{-7}$	< 0.001	< 0.1
15	$< 10^{-7}$	< 0.001	< 0.1
20	$< 10^{-7}$	< 0.001	< 0.1
25	$< 10^{-7}$	< 0.001	< 0.1
30	$< 10^{-7}$	< 0.001	< 0.1
35	$< 10^{-7}$	0.001	< 0.1
40	$< 10^{-7}$	0.002	< 0.1
45	$< 10^{-7}$	0.005	< 0.1
50	$< 10^{-7}$	0.010	< 0.1
55	$< 10^{-7}$	0.024	< 0.1
60	$< 10^{-7}$	0.064	< 0.1
65	$< 10^{-7}$	0.214	< 0.1
70	$< 10^{-7}$	0.968	0.1
75	0.000367	7.464	0.6
80	95.78	166.792	168.0
85	1841.0	5.000	3673.0
90	304.0	302.896	252.1

4.2.2 RESULTS FOR EXPONENTIAL SURFACE HEIGHTS

In Eq. (31) we indicated that the equivalent surface term for the exponential case had two elements. The J_S component, defined in Eq. (33), represents an incoherent scattered contribution, solely in the specular direction. As can be seen in Eq. (33), the term is independent of correlation function. The magnitude of J_S was calculated for our three Σ values, and for all cases it was orders of magnitude less than the associated J_D term for all three forms of the correlation function. This result also applies to other incident angles. As a result of this finding we assumed that $J_{EX} = J_D$ and made our comparisons using the results for J_D.

Since we wish to make additional comparisons in this section, the tables of results are more complicated than the previous set. The first comparison is the equivalent roughness effect for the three correlation functions. The second aspect is comparison of the results for the Gaussian surface height distributions with those for the exponential surface. Table 5 shows the corresponding patterns for both surfaces for a Gaussian correlation function and all three levels of roughness. Table 6 presents the same cases for a Bessel function correlation and Table 7 illustrates the results for a power law correlation function.

Consideration of the three tables shows that the broadening of the scattering pattern as the surface roughness increases for the exponential height cases is clearly present for all three correlation functions. The largest values occurred for Σ_2, for all cases. The power law and Gaussian results tended to be equivalent and slightly higher than the Bessel function correlation.

The comparisons for both surfaces with a given form of correlation also show very similar results for both cases. There appears to be a slight dominance for the exponential surface at the highest roughness condition but overall the scattering does not appear to be very sensitive to either correlation function or surface height statistics.

Table 5. Scattering Pattern Comparisons: Gaussian Correlation Function

θ_s	$J_D(\Sigma_1)$	$J_D(\Sigma_2)$	$J_D(\Sigma_3)$	$J_{GA}(\Sigma_1)$	$J_{GA}(\Sigma_2)$	$J_{GA}(\Sigma_3)$
0	0.278	$< 10^{-7}$	$< 10^{-7}$	0.169	$< 10^{-7}$	$< 10^{-7}$
5	0.439	$< 10^{-7}$	$< 10^{-7}$	0.365	$< 10^{-7}$	$< 10^{-7}$
10	0.681	$< 10^{-7}$	$< 10^{-7}$	0.714	$< 10^{-7}$	$< 10^{-7}$
15	1.042	$< 10^{-7}$	$< 10^{-7}$	1.287	$< 10^{-7}$	$< 10^{-7}$
20	1.581	$< 10^{-7}$	$< 10^{-7}$	2.176	$< 10^{-7}$	$< 10^{-7}$
25	2.390	$< 10^{-7}$	$< 10^{-7}$	3.495	$< 10^{-7}$	$< 10^{-7}$
30	3.610	$< 10^{-7}$	$< 10^{-7}$	5.392	$< 10^{-7}$	$< 10^{-7}$
35	5.473	$< 10^{-7}$	$< 10^{-7}$	8.061	$< 10^{-7}$	$< 10^{-7}$
40	8.36	4×10^{-7}	$< 10^{-7}$	11.77	$< 10^{-7}$	$< 10^{-7}$
45	12.91	84×10^{-7}	$< 10^{-7}$	16.93	$< 10^{-7}$	$< 10^{-7}$
50	20.28	0.000168	$< 10^{-7}$	24.14	$< 10^{-7}$	$< 10^{-7}$
55	32.59	0.003185	$< 10^{-7}$	34.42	$< 10^{-7}$	$< 10^{-7}$
60	53.99	0.058698	$< 10^{-7}$	49.51	61×10^{-7}	$< 10^{-7}$
65	93.23	1.068	$< 10^{-7}$	72.73	0.008710	$< 10^{-7}$
70	170.53	19.49	$< 10^{-7}$	110.90	3.146	$< 10^{-7}$
75	339.5	362.4	0.001134	180.3	308.6	0.000367
80	755.6	7011.	96.12	338.2	9319.	95.78
85	2155.0	47,646.	1827.	745.2	60,690.	1841.
90	6951.0	20,259.	303.	3007.	23,245.	304.

Table 6. Scattering Pattern Comparisons: Bessel Correlation Function

θ_s	$J_D(\Sigma_1)$	$J_D(\Sigma_2)$	$J_D(\Sigma_3)$	$J_{GA}(\Sigma_1)$	$J_{GA}(\Sigma_2)$	$J_{GA}(\Sigma_3)$
0	1.001842	0.7842256E-2	0.3644864E-3	1.251714	0.7539667E-2	0.4067682E-3
5	1.183998	0.1096810E-1	0.3395067E-3	1.394972	0.1043775E-1	0.3741979E-3
10	1.406118	0.1595884E-1	0.3332023E-3	1.555322	0.1500654E-1	0.3606812E-3
15	1.681536	0.2413276E-1	0.3560797E-3	1.740063	0.2237011E-1	0.3770124E-3
20	2.029083	0.3797172E-1	0.4256937E-3	1.958654	0.3459051E-1	0.4409223E-3
25	2.475949	0.6238065E-1	0.5738558E-3	2.223772	0.5562808E-1	0.5842757E-3
30	3.062382	0.1075915	0.8622235E-3	2.553009	0.9350622E-1	0.8686371E-3
35	3.849702	0.1962639	0.1418142E-2	2.971657	0.1655351	0.1421040E-2
40	4.934491	0.3819871	0.2523212E-2	3.517438	0.3119701	0.2522329E-2
45	6.474925	0.8009845	0.4851790E-2	4.248874	0.6352811	0.4844915E-2
50	8.742430	1.828352	0.1018411E-1	5.260902	1.426472	0.1016323E-1
55	12.23007	4.594681	0.2383641E-1	6.715843	3.628159	0.2377308E-1
60	17.89984	12.88892	0.6446788E-1	8.909580	10.80086	0.6424601E-1
65	27.81119	41.18707	0.2138344	12.42692	38.88502	0.2128606
70	46.96898	155.3386	0.9682934	18.55360	173.0694	0.9623992
75	90.05338	743.8073	7.463710	30.56559	964.2456	7.410263
80	217.0862	5501.954	166.4015	58.86668	7205.917	166.7919
85	635.9152	10,464.87	8.191567	151.4525	14,356.40	5.00030000
90	3272.934	18,948.55	302.2527	820.4220	22,157.13	302.8955

Table 7. Scattering Pattern Comparisons: Power Law Correlation Function

θ_s	$J_D(\Sigma_1)$	$J_D(\Sigma_2)$	$J_D(\Sigma_3)$	$J_{GA}(\Sigma_1)$	$J_{GA}(\Sigma_2)$	$J_{GA}(\Sigma_3)$
0	0.4931539	0.2577632E-2	0.	0.5445060	0.2621374E-2	0.
5	0.7163153	0.3097853E-2	0.	0.9115637	0.3133598E-2	0.
10	1.028297	0.3788023E-2	0.	1.434539	0.3869265E-2	0.
15	1.464896	0.4723974E-2	0.	2.150597	0.4767485E-2	0.
20	2.078585	0.6000929E-2	0.	3.104437	0.6149438E-2	0.
25	2.947735	0.7831130E-2	0.2778155E-2	4.353325	0.7979682E-2	0.3124718E-2
30	4.191814	0.1046576E-1	0.3516573E-2	5.974867	0.1073632E-1	0.0932072E-2
35	5.997374	0.1447467E-1	0.4495356E-2	8.079096	0.1491776E-1	0.4987461E-2
40	8.663810	0.2077193E-1	0.5880418E-2	10.82807	0.2100452E-1	0.6468201E-2
45	12.68723	0.3137043E-1	0.7881912E-2	14.46919	0.3150175E-1	0.8615432E-2
50	18.92176	0.5268517E-1	0.1088317E-1	19.39514	0.4486459E-1	0.1179106E-1
55	28.90776	0.1011426	0.1563782E-1	26.25854	0.4271110E-1	0.1666670E-1
60	45.58721	0.4765038	0.2375763E-1	36.20783	0.3850165E-1	0.2502987E-1
65	75.00725	4.769139	0.3889419E-1	51.41787	0.5546081	0.4049693E-1
70	130.8737	52.28599	0.7315932E-1	76.42770	27.13082	0.7526165E-1
75	248.8968	582.2431	0.6103984	122.0765	657.8823	0.5732417
80	546.4522	6654.589	167.6128	219.9775	8648.634	167.9552
85	1713.288	77,025.66	3639.308	498.9495	97,567.19	3672.704
90	5226.177	16,983.34	251.7047	2047.751	19,451.17	252.1492

4.3 Perturbation Regime

The preceding results and discussions are all for the case of a physical optics model where the heights are large compared to the wavelength. The third topic in this report is the extension of these considerations to the small scale height regime (heights small compared to the wavelength) where a perturbation method scattering cross section applies.

4.3.1 PERTURBATION METHOD

In this regime we have an alternative formulation for the normalized scattering cross section σ_{ss}. The conditions under which this relation can be applied are

$$k \sigma_s < 1 \tag{34}$$

and

$$\sigma_s / T_s < 1$$

where σ_s is the fine scale height variation, and T_s represents a correlation length for the small scale heights. The solution for $\sigma°_{ss}$ then is given by Ruck et al[1]:

$$\sigma°_{ss} = |\alpha_{pq}|^2 J_{ss} \tag{35}$$

with

$$J_{ss} = (2\pi) [(4k^4/\pi) \sigma_s^2 \cos^2 \theta_i \cos^2 \theta_s] \int_0^\infty c(\tau) J_o(|v_{xy}|\tau) \tau d\tau . \tag{36}$$

where

α_{pq} = scattering matrix elements for small scale of roughness [see Ruck et al[2]].

Note that the value of $\sigma°_{ss}$ is independent of the surface height statistics and depends only on the form of the correlation function. Thus the surface dependent results of the physical optics conditions do not apply here, and we are concerned only with the differences for the three types of correlation function as given in Eqs. (1), (2), and (24). The resulting integrals for $J_{ss}[c(\tau)]$ can be solved analytically and we have

(1) Gaussian correlation (c_1)

$$J_{ss} = [(4/\pi)k^4 \sigma_s^2 \cos^2\theta_i \cos^2\theta_s] [\pi T_s^2 \exp\{-T_s^2 v_x^2/4\}] . \tag{37}$$

(2) Bessel function correlation (c_2)

$$J_{ss} = [4k^2 T_s \sigma_s \cos\theta_i \cos\theta_s]^2 \left[\frac{3 v_x^4 T_s^4}{(1 + v_x^2 T_s^2)^4}\right] . \tag{38}$$

(3) Power law correlation (c_3)

$$J_{ss} = [8k^4 \sigma_s^2 \cos^2\theta_i \cos^2\theta_s] T_s^2 \exp\{-|v_x| T_s\} . \tag{39}$$

The variations of σ°_{ss} were studied in the same context as was done for the physical optics cases. We eliminated the dependence on the scattering matrix elements and make comparisons for the three forms of $J_{ss}[c(\tau)]$.

4.3.2 PERTURBATION RESULTS

As before, we chose $\lambda = 0.275$ m and $\theta_i = 85°$. Here $T_s = 0.0444$ m and we considered three levels of roughness, $k\sigma_s = 0.914$, 0.5, and 0.2 m. The results are presented in Tables 8, 9 and 10, respectively. The behavior is quite distinct from that of the large scale solutions. All three levels of roughness show similar angular distributions of scattered power and we will discuss the results together.

The Bessel function correlation showed somewhat different behavior from those of the power law and Gaussian cases. In all correlation cases, the relative magnitudes decreased with decreasing roughness. There is no tendency to peak near the specular direction as was true for large scale roughness. In contrast, all peaked at small θ_s with the Bessel function results being monotonic and the power law results always peaking at a slightly higher value of θ_s. The location of the peak was independent of roughness. The Gaussian and power law results have about the same magnitude (power law slightly higher) and the spread of the scattered power for both is fairly uniform up to $\theta_s = 60°$. The Bessel correlation, though, drops off rapidly with scattering angle θ_s and can be an order of magnitude below the peak when θ_s reaches 60°.

Table 8. Small Scale Scattering Angular Dependence, $k\sigma_s = 0.914$

θ_s	$J_{ss}(c_1)$	$J_{ss}(c_2)$	$J_{ss}(c_3$
0	0.202629E-01	0.196216E-01	0.1903339-01
5	0.209883E-01	0.192294E-01	0.206372E-01
10	0.213185E-01	0.178451E-01	0.220193E-01
15	0.212233E-01	0.155230E-01	0.230965E-01
20	0.206946E-01	0.125115E-01	0.237859E-01
25	0.197475E-01	0.922613E-02	0.240126E-01
30	0.184199E-01	0.613994E-02	0.237177E-01
35	0.167696E-01	0.363386E-02	0.228657E-01
40	0.148701E-01	0.188213E-02	0.214528E-01
45	0.128057E-01	0.837470E-03	0.195123E-01
50	0.106658E-01	0.312801E-03	0.171184E-01
55	0.854008E-02	0.949424E-04	0.143856E-01
60	0.651372E-02	0.223534E-04	0.114645E-01
65	0.466422E-02	0.376469E-05	0.853251E-02
70	0.305867E-02	0.392762E-06	0.578100E-02
75	0.175258E-02	0.187102E-07	0.339986E-02
80	0.789067E-03	0.174691E-09	0.156004E-02
85	0.198783E-03	0.140398E-11	0.397566E-03
90	0.	0.	0.

4.4 Composite Surface

The small scale surface results and the large scale height results have been presented as separate cases. As a final comment it should be noted that actual surfaces are often combinations of both types of height conditions. Ruck et al[2] have discussed how a composite, two scale surface may be obtained from the two solutions (as long as the slopes at the large scale roughness are small, $\sigma/T < 1$). The cross section of the small scale roughness [Eq. (35)] is simply added to that of the large scale roughness [Eq. (29)]. If the large scale surface slopes are not small, then the cross section for a composite, two scale surface is more complicated. This is discussed by Brown.[17]

17. Brown, G.S. (1978) Backscattering from a Gaussian distributed perfectly conducting rough surface, IEEE Trans. Antennas and Propag. APS-26:472-482.

Table 9. Small Scale Scattering Angular Dependence, $k\sigma_s = 0.5$

θ_s	$J_{ss}(c_1)$	$J_{ss}(c_2)$	$J_{ss}(c_3)$
0	0.606388E-02	0.587194E-02	0.569606E-02
5	0.628094E-02	0.575458E-02	0.617586E-02
10	0.637976E-02	0.534041E-02	0.658952E-02
15	0.635126E-02	0.464541E-02	0.691187E-02
20	0.619303E-02	0.374418E-02	0.711817E-02
25	0.590963E-02	0.276100E-02	0.718604E-02
30	0.551236E-02	0.183744E-02	0.709776E-02
35	0.501847E-02	0.108747E-02	0.684280E-02
40	0.445002E-02	0.563243E-03	0.641996E-02
45	0.383221E-02	0.250620E-03	0.583926E-02
50	0.319183E-02	0.936086E-04	0.512284E-02
55	0.255569E-02	0.284281E-04	0.430504E-02
60	0.194930E-02	0.668948E-05	0.343087E-02
65	0.139581E-02	0.112662E-05	0.255343E-02
70	0.915335E-03	0.117538E-06	0.173002E-02
75	0.524477E-03	0.559918E-08	0.101744E-02
80	0.236135E-04	0.522780E-10	0.466857E-03
85	0.594878E-04	0.420153E-12	0.118975E-03
90	0.	0.	0.

Table 10. Small Scale Scattering Angular Dependence, $k\sigma_s = 0.2$

θ_s	$J_{ss}(c_1)$	$J_{ss}(c_2)$	$J_{ss}(c_3)$
0	0.970216E-03	0.939511E-03	0.911373E-03
5	0.100495E-02	0.920733E-03	0.988138E-03
10	0.102076E-02	0.854467E-03	0.105432E-02
15	0.101620E-02	0.743264E-03	0.110590E-02
20	0.990889E-03	0.599070E-03	0.113890E-02
25	0.945540E-03	0.441760E-03	0.114976E-02
30	0.881973E-03	0.293990E-03	0.113564E-02
35	0.802953E-03	0.173995E-03	0.109485E-02
40	0.712005E-03	0.901194E-04	0.102719E-02
45	0.613154E-03	0.400993E-04	0.934280E-03
50	0.510693E-03	0.149774E-04	0.819653E-03
55	0.408911E-03	0.454849E-05	0.688806E-03
60	0.311888E-03	0.107032E-05	0.548939E-03
65	0.223330E-03	0.180259E-06	0.408549E-03
70	0.146454E-03	0.188259E-07	0.276804E-03
75	0.839161E-04	0.895869E-09	0.162790E-03
80	0.377816E-04	0.836447E-11	0.746971E-04
85	0.951799E-05	0.672242E-13	0.190360E-04
90	0.	0.	0.

5. CONCLUSIONS

This report has been concerned with the relationship between surface height statistics and correlation and the scattering of em waves by the surface. The first analysis showed that establishing conditions on the correlation length ($T >> \lambda$) was sufficient to cause the radius of curvature to be large compared to the wavelength as well. Hence this allows us to establish the conditions for which physical optics principles can be used to analyze the scattering from the random rough surface. The second major theme of the report examined the manner in which the pattern of scattered power as a function of the surface roughness is dependent on the surface height statistics and the form chosen for the correlation function of those surface heights.

The first topic involved the determination of limits for the application of the physical optics scattering cross section model. The first restriction in the analysis is that the surface height distribution be Gaussian. Under this constraint it was shown that the surface slopes and slope derivatives are statistically uncorrelated. Then, for surfaces with either a Gaussian or power law correlation function, we conclude that for intermediate or large slopes the condition $T >> \lambda$ is both necessary and sufficient for $R_c >> \lambda$. That is, if one wishes to apply physical optics in those regimes, one is restricted to the class of surfaces for which $T >> \lambda$. On the other hand, for small slope regimes the applicability is less restrictive. There, $T >> \lambda$ is still a sufficient condition for $R_c >> \lambda$ but it is no longer necessary. Surfaces whose correlation length is on the order of wavelength ($T \approx \lambda$) can also generate the condition $R_c >> \lambda$. A further aspect of this is that since either T or λ can be varied there is no particular restriction on the value of the Rayleigh parameter in the various slope regimes.

In the second topic addressed in this report, a study is made of three aspects of the angular dependence of rough surface scattering. The results assess the dependence on surface height statistics and surface correlation function, but do not include the effects of surface dielectric constant in the scattering matrix component of the scattering cross section. For physical optics conditions, the results show that for a Gaussian surface height distribution, the scattering angle dependence for all three correlation functions is similar. For a smooth surface the contributions are mainly in the specular direction and the pattern becomes broader as surface roughness increases. The Gaussian and power law correlation results are about the same, while the Bessel correlation results in more variability. The magnitude is greatest for intermediate roughness. Next, the dependence of surface statistics was examined. For the case of exponential surface heights, there is an additional diffuse contribution in the specular direction, independent of correlation function; for the cases considered, this term is always less than the general diffuse scattering term. The resulting distributions repeated the roughness dependence of the Gaussian surface. Also, the Gaussian and power law results are similar and the Bessel correlation is slightly different. For instance, at $\theta_i = 85°$, it results in a null near the specular direction for the smallest roughness case. When the analysis was extended to perturbation method regimes (heights small compared to a wavelength) the behavior is quite different. In that case, the narrowing of the scattering pattern with decreased roughness is not present. For all three correlations, the magnitudes decrease with decreasing roughness but the individual patterns are preserved. The Gaussian and power law patterns are broad (up to $\theta_s = 60°$) while the Bessel function correlation shows a rapid falloff with increasing scattering angle θ_s; all three have their largest values for small θ_s. To summarize, the only factor that introduced any variability into the results was the use of the Bessel function correlation and, even for that case, the trends are similar.

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